

STX

1132 COPY 2



BEBR

**FACULTY WORKING
PAPER NO. 1132**

THE LIBRARY OF THE

APR 24 1983

UNIVERSITY OF ILLINOIS
URBANA-CHAMPAIGN

An Empirical Analysis of Unitary Apportionment

Karen S. Hreha

Peter A. Silhan

BEBR

FACULTY WORKING PAPER NO. 1132

College of Commerce and Business Administration

University of Illinois at Urbana-Champaign


March, 1985

An Empirical Analysis of Unitary Apportionment

Karen S. Hreha, Assistant Professor
Department of Accountancy

Peter A. Silhan, Assistant Professor
Department of Accountancy

We gratefully acknowledge the financial support of Coopers & Lybrand.



Digitized by the Internet Archive
in 2011 with funding from
University of Illinois Urbana-Champaign

<http://www.archive.org/details/empiricalanalysisi1132hreh>

ABSTRACT

This study investigates the effects of alternative factor weightings on unitary apportionment. The results indicate that the payroll factor tends to distort income allocations. Allocative fairness could be enhanced by eliminating the payroll factor and using a sales-property formula instead.

AN EMPIRICAL ANALYSIS OF UNITARY APPORTIONMENT

INTRODUCTION

The out-of-state activities of a business cannot be taxed by a state unless (a) there is some connection between its out-of-state activities and the taxing state, and (b) there is a rational relationship between the income attributed to the state and the intrastate values of the enterprise [Mobil Oil Corp. v. Commissioner, 445 U.S. 436, 437 (1980)].

To satisfy condition (a), a company must operate as a unitary business [Mobil, 445 U.S. 439 (1980)]. The legal test for this requires that contributions to income by subsidiaries result from functional integration, centralization of management, and economies of scale [Mobil, 445 U.S. 438]. The New Mexico Supreme Court has ruled that "control over subsidiaries, the interdependence between parent and affiliates, the history of relationships, and the placing of funds on a general account, all point to functional integration and reflect the existence of an underlying unitary business" [Taxation and Revenue Dept. v. F. W. Woolworth Co., 95 N.M. 519, 529, 624 P.2d 28, 29 (1981)]. The U.S. Supreme Court has ruled that no single factor is considered sufficient to establish the existence of a unitary business [ASARCO Inc., etc. v. Idaho State Tax Commission, 102 S. Ct. 3103 (1982); F. W. Woolworth Co. v. Taxation and Revenue Dept. at the State of New Mexico, 102 S. Ct. 3128 (1982)].

To satisfy condition (b), some cause and effect relationship must be demonstrated. This condition, which affects allocative fairness, is the focus of this study.

UNITARY APPORTIONMENT

Forty-five states, out of the forty-six states which levy corporate income taxes, have adopted the so-called three factor formula [Hellerstein, 1983, p. 495]. With this formula, unitary corporations allocate business income among the states by averaging three proportions: in-state sales to total sales, in-state property to total property, and in-state payroll to total payroll. The underlying variables are defined in the Uniform Division of Income for Tax Purposes Act [UDITPA].¹ Business income, which is net income before federal taxes, is generated in the ordinary course of operations. It includes income from tangible and intangible property [UDITPA § 1(a)].

The three-factor formula can be denoted as follows:

$$\hat{y}_j = Y[a_1 (\frac{s_j}{\sum s_j}) + a_2 (\frac{c_j}{\sum c_j}) + a_3 (\frac{w_j}{\sum w_j})] \quad (1a)$$

where

\hat{y}_j = income apportioned to state j,

Y = total business income

$a_i = 1/3$ (i = 1,2,3)

s_j = sales attributable to state j,

c_j = property attributable to state j,

w_j = payroll attributable to state j,

or, equivalently

$$\hat{y}_j = Y\hat{p}_j \quad (1b)$$

$$\hat{p}_j = [a_1 p_{1j} + a_2 p_{2j} + a_3 p_{3j}] \quad (2)$$

where

\hat{p}_j = proportion of income attributable to state j,
 p_{1j} = proportion of sales attributable to state j,
 p_{2j} = proportion of property attributable to state j,
 p_{3j} = proportion of payroll attributable to state j.

The sales factor includes gross receipts from services, rentals, royalties, and business operations [UDITPA § 1(g)]. It encompasses sales, less returns and allowances, plus all interest income, and service and carrying charges incidental to such sales. The destination rule for receipts from sales of tangible personal property has spread rapidly and is now in use in forty of the forty-five states that have a sales factor in their apportionment formula. Identification of the destination of a sale is the state in which the property is delivered or shipped to the purchaser, regardless of the f.o.b. point or other conditions of sale [UDITPA § 16(a)].

The property factor includes the average value of all real and tangible personal property used in the regular course of the trade or business. In general, properties consist of land, buildings, machinery, inventory, and equipment. Property held as reserves or standby facilities is includible; however, property held solely for investment is not. Traditionally, this factor was restricted to property owned by the taxpayer, but in recent years the rule has been modified by most states to include rented property used in the business. Rental property generally is valued at eight times the net rental cost (defined as the annual rental rate less rentals from subleases of the property). Most states use original cost to value other tangible assets for apportionment purposes [UDITPA § 11]. Beginning and ending values are averaged.

The payroll factor is defined as wages, salaries, commissions and any other form of remuneration paid to employees for personal services [UDITPA § 1(c)]. The compensation of an employee is attributed to a particular state if the services are performed entirely within the state. If the services are performed both within and without the state, payroll is attributable to the state where the operations are based.

RESEARCH OBJECTIVES

The goal of unitary apportionment is to allocate taxable income among the states in a manner reasonably consistent with how corporate income is generated [Container Corp. v. Franchise Tax Bond, 103 S. Ct. 2933 (1983)]. Unitary apportionment implies that allocative fairness, condition (b), can be satisfied with a simple formula [NTA Proc. 1950, p. 349]. The relationship between the three apportionment factors and income can be viewed as one of implied causality whereby the factors are each presumed to contribute to business income, the dependent variable.²

In this study we empirically analyze the allocative fairness of the three-factor formula vis-a-vis a number of simpler alternatives. No previous study has addressed this issue.³ It is shown that the payroll factor tends to distort income allocations and that several simpler models provide more accurate predictions. Separate accounting, which has served as a legal benchmark, is used to measure allocative fairness.⁴ The Committee of the National Tax Association on the Allocation of Income and the American Institute of Accountants concluded that separate accounting is theoretically preferable to fractional apportionment [NTA Proc. 1939, p. 195].

METHODOLOGY

Unitary apportionment is necessary because most large corporations operate across state boundaries. Corporations which operate in just one state do not have to allocate income for state tax purposes.

Ideally, there would be a one-to-one correspondence between taxable profits by segment and taxable profits by state. That is, for each segment or line of business, only a single state would be involved. This ideal, depicted in Figure 1, implies that separate accounting would be preferred as long as each segment were treated as a separate profit center. Separate accounting would also provide a benchmark for evaluating alternative attribution formulas on a pro forma basis.

However, when firms operate across jurisdictional boundaries, profitability by state becomes more difficult to measure and formula apportionment becomes necessary to estimate taxable income by jurisdiction. As Figure 2 implies, however, this jurisdictional shift does not affect the underlying economics of the firm. A firm's profit function is not affected by jurisdictional realignments.

By separating the economic and jurisdictional determinants of profit, it is possible to evaluate the three-factor formula at several levels. As Figure 2 implies, it could be evaluated at the segment level. However, since the same formula applies to all corporations, it could also be evaluated at the corporate level (see Appendix). Indeed, the same relationships must hold across company and jurisdictional boundaries. Moreover, a segment would use the same formula as a company would if it were merged with another company. This consistency

makes it possible to use corporate data for purposes of testing the three-factor formula.⁵

General Model

Equation (2) was operationalized in terms of a regression as follows:

$$p_k = b_1 p_{1k} + b_2 p_{2k} + b_3 p_{3k} + e_k \quad (3)$$

where

p_k = proportion of total income associated with firm k;

p_{1k} = proportion of total sales associated with firm k;

p_{2k} = proportion of total property associated with firm k;

p_{3k} = proportion of total payroll associated with firm k;

b_i = factor coefficients;

e_k = error term.

To evaluate different formulas, eleven profit attribution models were tested within a holdout sample. These models were evaluated in terms of their ability to predict the proportion of total income associated with firm k. This estimation-validation approach provided a pure test of the formula weights per se.

Evaluative Approach

Allocative fairness, condition (b), was evaluated in six phases. First, ordinary least squares (OLS) and restricted least squares (RLS) regressions were used to estimate the factor coefficients of the three statutory factors. These regression weights were then compared to the

current weights. Second, the OLS and RLS formulas were used to predict income proportions in a three-year holdout period. These income predictions were compared with corresponding predictions based on the original formula. Third, in order to evaluate the effects of excluding a factor, two-factor models were estimated. Fourth, two-factor regressions were used to predict income and errors were compared to those of the original formula. Fifth, the predictive ability of two-factor regressions was compared to the predictive ability of two-factor models with equal weighting. Sixth, the comparative accuracy of all eleven models was determined using rank-order statistics.

Error Metrics

All things being equal, an apportionment method with less error would be considered fairer than an apportionment method with more error. Alternative formulas were thus evaluated in terms of margins of error. This is consistent with the approach used by the courts.

To evaluate allocative fairness, margins of error were computed within a post-estimation holdout sample (1979-81). This out-of-sample approach has been used by economists and others to validate a variety of causal relationships.⁶

Two metrics were used to assess allocative fairness. Mean absolute relative error (MARE) and mean absolute error deflated by mean earnings (MAE) were defined as follows:

$$MARE = \frac{1}{N} \sum_{k=1}^N \sum_{t=79}^{81} \left| \frac{\hat{p}_{kt} - p_{kt}}{p_{kt}} \right|$$

$$MAE = \frac{1}{N} \sum_{k=1}^N \sum_{t=79}^{81} \left| \frac{\hat{p}_{kt} - p_{kt}}{\bar{p}_t} \right|$$

• where

$$\begin{aligned} k &= \text{Number of firms indexed by } k; \\ \hat{p}_{kt} &= \text{Predicted income proportion for firm } k \text{ in period } t; \\ p_{kt} &= \text{Actual income proportion for firm } k \text{ in period } t; \\ \bar{p}_t &= k^{-1} \sum_{k=1}^k p_{kt}. \end{aligned}$$

In effect, the first metric treats each firm as having equal importance, while the second one treats each dollar of error as having equal importance. The first can be viewed as operationalizing fairness on a firm-by-firm basis, while the second can be viewed as operationalizing fairness on a societal level.

DATA SAMPLE

Because corporate tax returns are generally kept confidential, we could not use such data to analyze the current formula. We could, however, use financial accounting data. This approach has been used by Sheffrin and Fulcher [1983] to examine the worldwide unitary method of state taxation.

Zimmerman [1983] has compared financial accounting data to tax data. He found that statistical differences were not significant. Computations of corporate tax rates based on COMPUSTAT financial data were comparable to corporate tax rates based on IRS data. Magnitudes, trends, and cross-sectional differences were analyzed empirically.

He concluded that financial data appeared to yield unbiased estimates of effective tax rates.

Sampled Firms

A sample of 102 COMPUSTAT companies was used to empirically estimate the relationship between the statutory factors and income. Each firm in the sample provided sales, property, payroll, and income data for twelve consecutive years ending with 1981. In addition, another 78 firms were added to this sample to form a larger sample for validating the models in terms of post-estimation performance.

The three-factor formula was originally designed for manufacturing and mercantile businesses [NTA Proc. 1939, p. 195]. Different methods of apportionment are applied in many but not all states to electric and gas utilities, transportation and communication businesses (which are often taxed on their gross receipts), and insurance companies (which are typically taxed on the basis of their gross premiums, Hellerstein, 1983, pp. 632-685). For this reason, utilities, transportation, communication, and financial companies were excluded from the sample.

Variables of Interest

Whenever data surrogates are used, care must be taken to ensure that the surrogates do, in fact, represent reasonably well the underlying concepts and constructs of interest. Variables in this study were therefore examined for representational faithfulness.

Although states have a variety of income definitions for state tax purposes, there is broad conformity to the Federal income tax base

(Hellerstein, 1983, p. 266). This tends to reduce differences among states.

In general, differences between state taxable income and Federal taxable income involve the treatment of capital gains and net operating losses. These differences, however, tend to move state taxable incomes closer to accounting incomes than Federal tax incomes. Most states do not allow preferential treatment for capital gains and do not allow net operating loss carrybacks and carryforwards.

Accounting income before Federal income taxes was used in this study because few states allow a deduction for Federal income taxes. No adjustments were made to the accounting data for dividends received because deductions for such dividends are not generally made by the states.

A major difference between state taxable income and accounting income involves depreciation. Most states allow accelerated depreciation, but most corporations use straight-line depreciation. This difference, however, along with other minor differences, is mitigated by the fact that this study relies on cross-sectional data. Differences that affect all firms in basically the same way tend to cancel out. Consequently, financial accounting information can be used in place of tax information to investigate the allocative fairness of various profit attribution formulas.

The following COMPUSTAT variables were used to provide accounting data:

Sales - Item 12 (net sales)

Property - Item 3 (inventory) plus
- Item 7 (gross plant) plus 8 times
Item 47 (rental expense)

Payroll - Item 42 (labor and related expense)

Income - Item 13 (operating income before depreciation) minus
Item 14 (depreciation) minus
Item 15 (interest expense).

To compute the property factor, the beginning and ending values for inventory and gross plant were averaged in conformance with UDITPA. Rental property was valued at eight times the net rental expense. Payroll included all forms of remuneration for personnel services.

EMPIRICAL RESULTS

The results presented here indicate that the weights currently assigned to the three statutory factors tend to inflate apportionment errors.

Phase 1: Three-Factor Regressions

Separate regressions were performed for each of the twelve years comprising the estimation sample (1967-78). Each regression was based on a common sample of 102 firms. RLS and OLS regressions were conducted in parallel in order to evaluate the possibility of omitted variables.

Table 1 compares the computed regression weights to the current one-third weights. It indicates that the current weights did not even approximate the regression weights in either sign or magnitude. Only the property factor was remotely close to having a one-third coefficient. The sales factor, by far, dominated the other two factors in terms

of magnitude. As expected, the adjusted coefficients of determination, \bar{R}^2 , indicated that the equally-weighted formula did not fit the estimation sample as well as the regression weights.

Figure 3 shows how the three-factor coefficients behaved over time. While there are fluctuations from year to year, the relative position of the factors remains intact. This pattern suggests that multicollinearity was not harmful. "It is well known that collinearity need not harm forecasts, even if it has harmed structural estimation, as long as it continues into the forecast period" [Belsley, 1984, p. 183]. Furthermore, multicollinearity is often not a problem when the purpose of the regression analysis is to make predictions of new observations within the range of observations [Neter and Wasserman, 1976, p. 345].

Table 2 provides additional evidence concerning the means, standard deviations, and correlations among the factors and income. The highest collinearity is between sales and property.

Phase 2: Three-Factor Performance

The predictive ability of the equally-weighted model versus the regression-weighted model was tested using COMPUSTAT data for years 1979-81. Predictions from these models were compared by computing mean errors in the holdout sample. As shown in Table 4, the mean absolute relative error resulting from the statutory formula is close to 90.6 percent. This error is reduced to 64.6 percent when the unequally-weighted regression model is used. When the mean absolute error is deflated by mean earnings, the error produced by the statutory formula is 51.3 percent as compared to 39.4 percent when the three-factor

regression model was used (Table 5). Thus, the three-factor regression model significantly reduces distortion of income allocation within an individual firm and also within a given state.

Phase 3: Two-Factor Regressions

In order to evaluate the possibility that the value of one of the coefficients might, in fact, be zero, two-factor regressions were also performed. Table 3 shows that, on average, the payroll factor had a negative weight regardless of whether sales or property was the other factor. When the payroll factor was removed, the regression coefficients for sales and property were essentially equal. This tends to confirm that collinearity exists between these factors; that is, the sales and property proportions appear to be essentially redundant. Table 2 shows that the correlation coefficient between these factors was .9174. Furthermore, the correlation between sales and income was .8784 and the correlation between property and income was .8841.

Phase 4: Two-Factor Performance

Margins of error for the two-factor models are presented in Tables 4 and 5. Relative to the current formula, only the sales-payroll combination with equal weighting and the property-payroll combination with equal weighting did worse in terms of MAREs and MAEs. The other two-factor combinations outperformed the current three-factor formula.

When margins of error were measured in terms of MAREs, the sales-payroll regression performed best among the two-factor models. It produced a mean error that was 27.0 percent lower than the existing formula. However, when errors were measured in terms of MAEs, the

sales-property regression performed best. It reduced the margin of error by 24.4 percent from the current formula.

These results suggest that the payroll factor should not have a positive weight. This implies that the payroll factor might be the prime contributor to inflated errors. In fact, both the two- and three-factor models with positive payroll weights performed poorly.

Phase 5: One-Factor Performance

One-factor models were also evaluated for their predictive ability.⁷ Tables 4 and 5 indicate that only the payroll model performed worse than the current formula. The sales and the property factors each performed better than the current formula, while the payroll weight tended to inflate the margin of error.

Phase 6: Formula Rankings

In the final phase, the forecasting performance of all eleven models was ranked in order to provide a different view of how much the margins of error were affected by the weightings per se. Friedman rank sums [Hollander and Wolfe, 1973, p. 138] indicated that there were significant differences among the models in terms of mean ranks. For each firm the best performing model was assigned a value of one, while the worst was assigned a value of eleven. Intermediate values were assigned in order. Table 6 shows that model (3), the sales-property combination with equal weighting, outperformed the others in terms of rankings. It is interesting, however, that the sales factor alone provided reasonably accurate predictions of income.

CONCLUSIONS

The results of this study suggest that less income distortion would occur if the statutory factors were weighted differently. The RLS weights appear to capture more accurately the underlying relationship between the statutory factors and corporate income. This is reflected in both model fit and predictive ability. However, these coefficients also indicate that payroll is consistently negative when used with sales and property. Thus, if labor were the dominant factor in a state, the income apportioned to that state under the current formula would be grossly overstated.

This finding also affects those states which apply the three-factor formula to worldwide income. Since domestic wage rates are typically higher than foreign wage rates, domestic income would be overstated even more than it is now. This overstatement increases the potential for double taxation. The results reported here therefore apply to the related issue of double taxation which has generated much controversy in recent years.

If the payroll factor were eliminated, a sales-property model would perform well. The equally-weighted sales-property model was ranked best overall in terms of predictive ability. This model would also simplify, to some extent, state tax determination.

Footnotes

¹For purposes of this study, the UDITPA definitions are used. These definitions have been adopted by twenty-three states and the remaining twenty-two states which use three-factor apportionment have adopted similar definitions which are essentially the same in concept.

²See Granger [1969] and Newbold [1982] for an overview of causality testing.

³In 1964, a Special Subcommittee on State Taxation compared alternative formulas in terms of their effects on selected companies and states. Its methodology, however, did not address the issue of allocative fairness per se. No attempt was ever made to empirically estimate and evaluate the three-factor formula as such.

⁴In Hans Rees' Sons, Inc. v. North Carolina ex rel Maxwell [283 U.S. 123 (1931)], an apportionment method based entirely on tangible property resulted in an attribution to North Carolina of approximately 76% of the taxpayer's income over several years compared to separate accounting which resulted in an attribution of 21.7%. The Supreme Court struck down the use of the one-factor formula because of this large difference. In Container, the percentage increase in taxable income attributable to California by the three-factor apportionment method as compared to the separate accounting method was only 14%. The Supreme Court considered that figure acceptable [Container, p. 2950].

⁵Conceptually, the approach used here is similar in many respects to the simulated-merger approach that has been used to assess the effects of data aggregation on predictions of conglomerate earnings [Silhan, 1982]. In this study, however, the focus is on the sub-entity level as opposed to the total-entity level.

⁶See, for example, Ashley, Granger, and Schmalensee [1980].

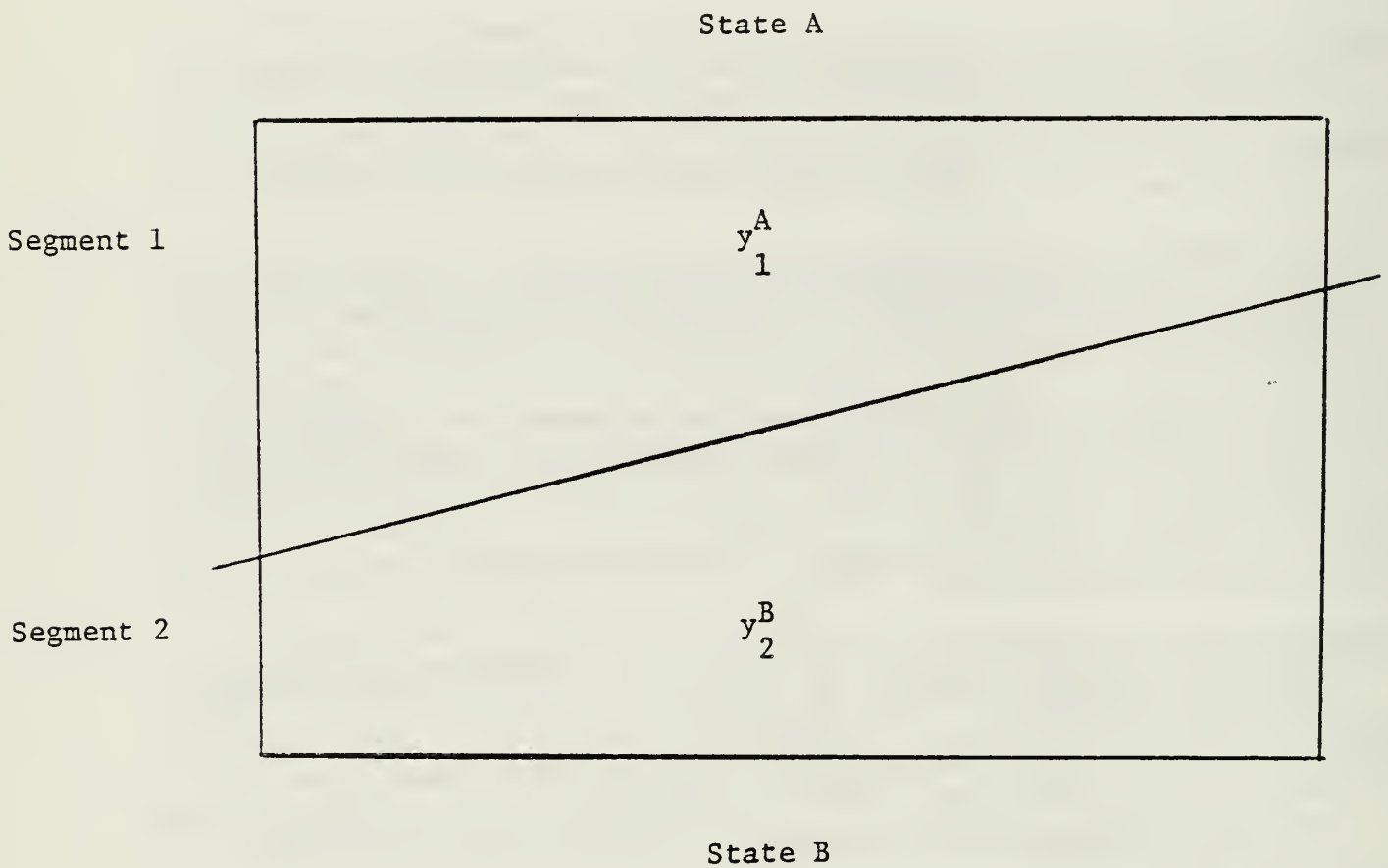
⁷The factor coefficients in these models were set equal to one with no intercept.

References

- Ashley, R., C. W. J. Granger, and R. Schmalensee, "Advertising and Aggregate Consumption: An Analysis of Causality," Econometrica 48 (1980), pp. 1149-67.
- Belsley, D. A., "Collinearity and Forecasting," Journal of Forecasting (April-June 1984), pp. 183-96.
- Granger, C. W. J., "Investigating Causal Relations by Econometric Models and Cross-Spectral Methods," Econometrica (July 1969), pp. 424-38.
- Hellerstein, Jerome R., State Taxation, Vol. I, Corporate Income and Franchise Taxes (Warren, Gorham & Lamont, 1983).
- Hollander, M. and D. A. Wolfe, Nonparametric Statistical Methods (John Wiley & Sons, Inc., 1973).
- National Tax Association, "Second Preliminary Report of the Committee on Tax Situs and Allocation," Proceedings of Forty-Third National Conference (1950), pp. 349-66.
- _____, "Report of the Committee of the National Tax Association on Allocation of Income," Proceedings of Thirty-Second National Conference (1939), pp. 190-232.
- Neter, J., and W. Wasserman, Applied Linear Statistical Models (Richard D. Irwin, Inc., 1976).
- Newbold, P., "Causality Testing in Economics," Time Series Analysis: Theory and Practice (1982), pp. 701-716.
- Sheffrin, Steven M. and Jack Fulcher, "The Container Case: Can We Identify the Winners and Losers," Proceedings of the Seventy - Sixth Annual Conference (1983), National Tax Association, pp. 33-39.
- Silhan, P. A., "Simulated Mergers of Existent Autonomous Firms: A New Approach to Segmentation Research," Journal of Accounting Research 20 (Spring 1982), pp. 255-262.
- State Taxation of Interstate Commerce (1964), Report of the Special Subcommittee on State Taxation of the Committee on the Judiciary, 88th Congress, 2nd Session, House Report No. 1480.
- Zimmerman, J. L., "Taxes and Firm Size," Journal of Accounting and Economics (August 1983), pp. 118-149.

FIGURE 1

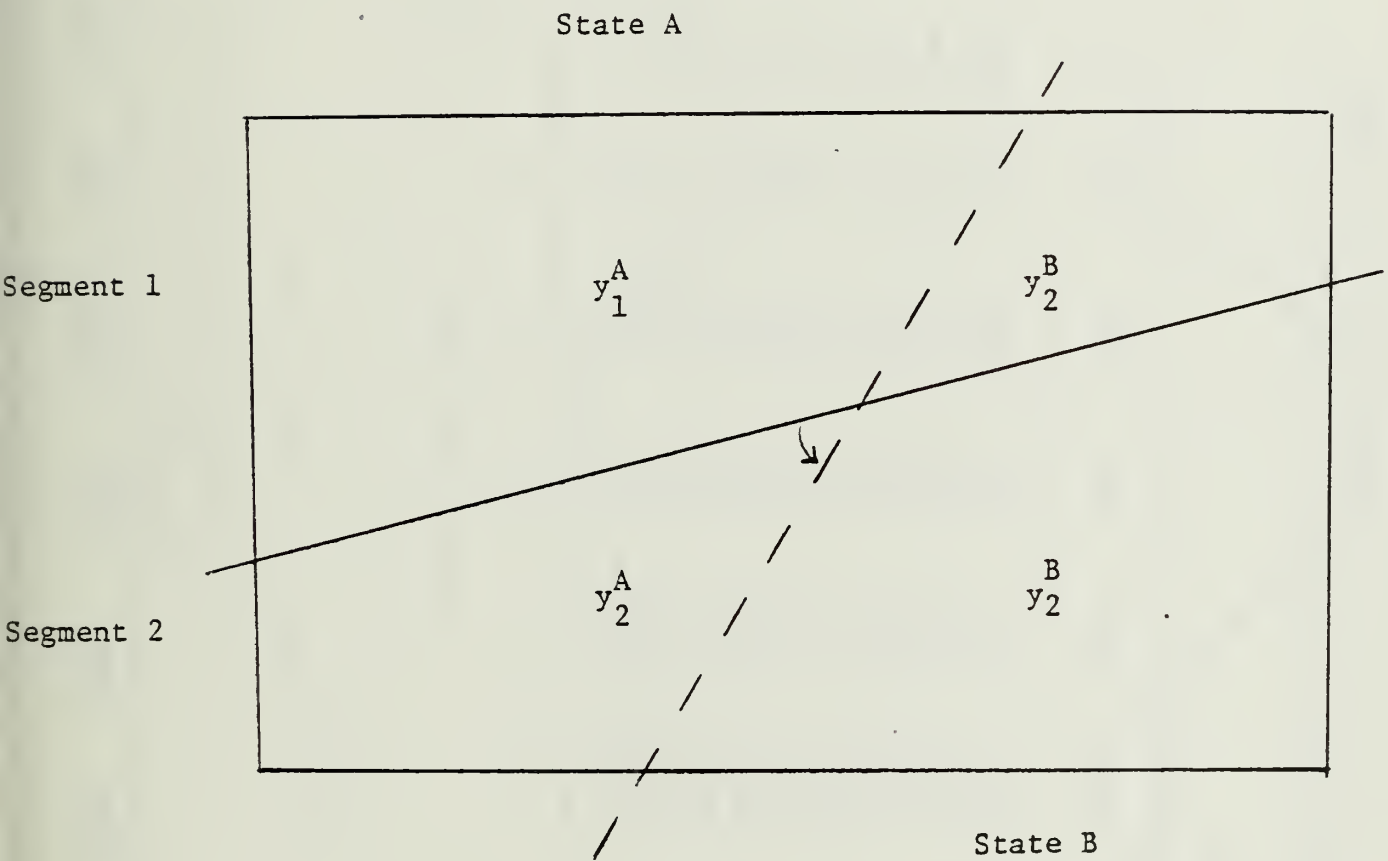
Unitary Profits with One-to-One
Correspondence between Segments and States
(Separate Accounting)



y_i^s = Profit attributed to Segment i and State s

FIGURE 2

Unitary Profits with Many-to-Many
Correspondence between Segments and States
(Apportionment)



y_i^s = Profit attributed to Segment i and State s

FIGURE 3

Three-Factor Coefficients

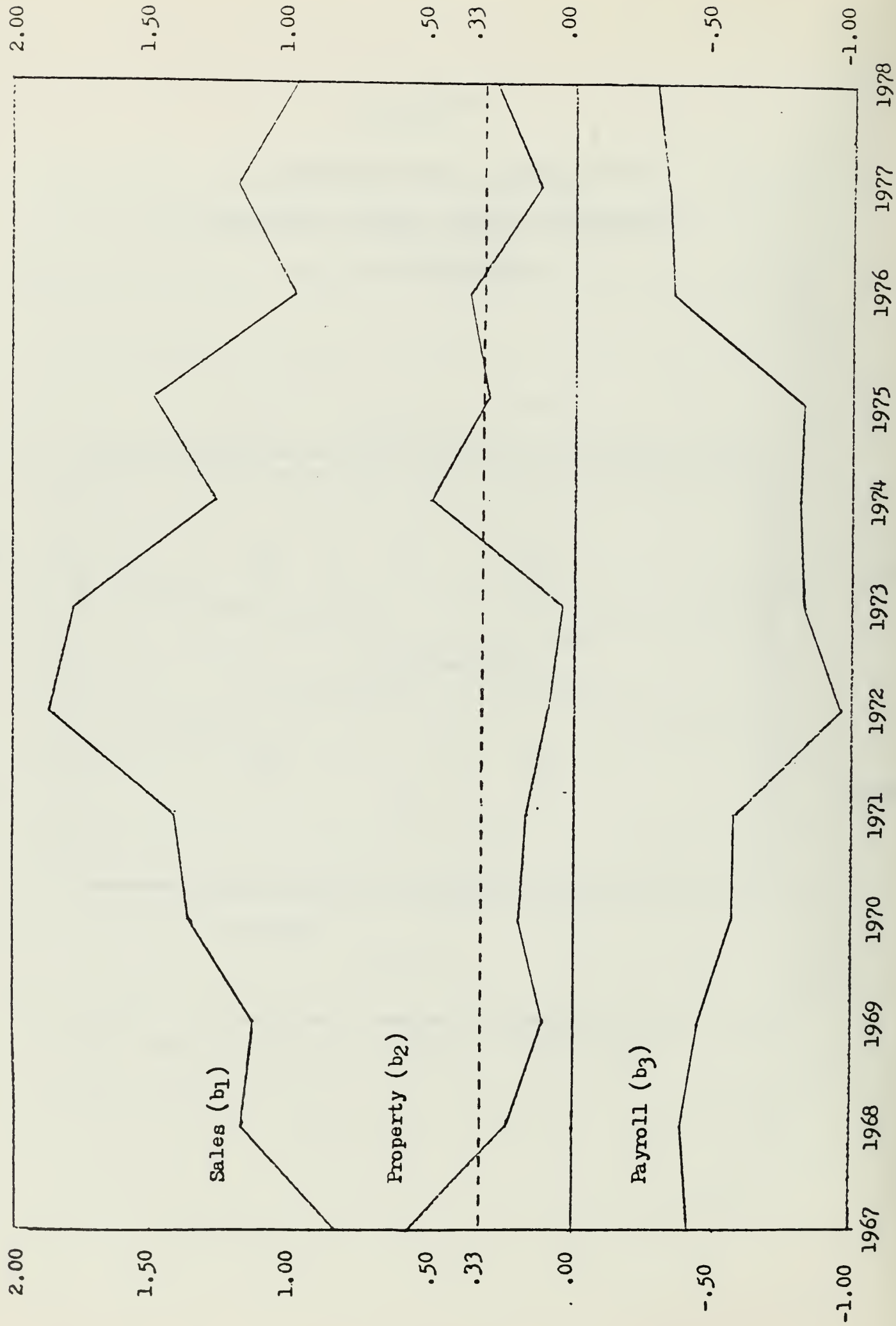


TABLE 1
Three-Factor Coefficients

Year	Current Formula (RLS)				Unconstrained (OLS)				Constrained (RLS) ^a			
	b_1	b_2	b_3	R^2	b_0	b_1	b_2	b_3	R^2	b_1	b_2	b_3
1967	.3333			.627	.0013	.8453	.5292	-.5092	.860	.8553	.5709	-.4262
1968	.3333			.807	.0000†	1.1719	.2279	-.3995	.933	1.1720	.2277	-.3997
1969	.3333			.704	.0005†	1.1429	.1230	-.4450	.911	1.1316	.2747	-.4063
1970	.3333			.724	.0002†	1.3691	.1914	-.5766	.925	1.3648	.2006	-.5654
1971	.3333			.743	-.0005†	1.3913	.1932	-.5302	.929	1.4147	.1567	-.5714
1972	.3333			.698	-.0011†	1.7878†	.1944	-.8744	.916	1.8676	.0936†	-.9613
1973	.3333			.603	-.0008†	1.7409	.1078	-.7654	.935	1.7900	.0433†	-.8333
1974	.3333			.501	-.0013†	1.1752	.6719	-.7133	.845	1.2891	.5156	-.8047
1975	.3333			.490	-.0006†	1.4599	.3787†	-.7794	.803	1.5074	.3143†	-.8217
1976	.3333			.663	-.0002†	.9699	.3917†	-.3416	.865	.9886	.3662	-.3548
1977	.3333			.708	-.0003†	1.1594	.1786†	-.3077	.888	1.1933	.1360†	-.3293
1978	.3333			.704	.0001†	1.0131	.2737	-.2995	.874	.9996	.2907	-.2903
1967-78	.3333			.648	-.0002†	1.2910	.2731	-.5402	.866	1.3025	.2551	-.5575
												.866

† = not significant at .05 level

OLS = ordinary least squares

RLS = restricted least squares

b_0 = intercept; b_1 = sales factor; b_2 = property factor; b_3 = payroll factor

^aAn unconstrained intercept yielded essentially the same factor weightings

TABLE 2

Means, Standard Deviations, and Zero-Order Correlations
(1967-78)

	Mean	SD	Pearson Coefficients		
			Sales	Property	Payroll
Sales	.0098	.0166	—	--	--
Property	.0098	.0170	.9174	--	--
Payroll	.0098	.0160	.7439	.5683	--
Income	.0098	.0220	.8784	.8841	.4541

TABLE 3

Two-Factor RLS Regression Coefficients

Year	Sales-Property			Sales-Payroll			Property-Payroll		
	b_1	b_2	\bar{R}^2	b_1	b_3	\bar{R}^2	b_2	b_3	\bar{R}^2
1967	.1445†	.8555	.811	1.6790	-.6790	.804	1.0551	-.0513†	.809
1968	.5403	.4597	.903	1.5169	-.5169	.925	.8365	.1635	.854
1969	.4781	.5219	.873	1.5356	-.5356	.897	.8877	.1123	.832
1970	.4536	.5464	.857	1.6407	-.6407	.919	.9943	.0057†	.822
1971	.4889	.5111	.863	1.6282	-.6282	.924	.9914	.0086†	.827
1972	.4443	.5557	.826	1.8519	-.8519	.931	1.0424	-.0424†	.798
1973	.0538†	.9462	.782	1.9966	-.9966	.911	1.2739	-.2739	.816
1974	.0875†	.9125	.676	1.8250	-.8250	.835	1.6718	-.6718	.811
1975	.7619	.2381†	.644	1.8182	-.8182	.799	1.6751	-.6751	.745
1976	.6345	.3655	.810	1.3547	-.3547	.857	1.2441	-.2441	.818
1977	.7378	.2622	.843	1.3377	-.3377	.886	1.1215	-.1215	.814
1978	.5718	.4282	.838	1.3150	-.3150	.867	1.0942	-.0942	.817
1967-78	.4324	.5676	.787	1.6067	-.6067	.860	1.1528	-.1528	.781

† = not significant at .05 level

RLS = restricted least squares estimation

 b_1 = sales factor; b_2 = property factor; b_3 = payroll factor

TABLE 4

Mean Absolute Relative Error (1979-81)

Statutory Factors	Formula	Weighting	Error ^a		Change ^b		Significance ^c	
			Mean	(SD)	Mean	%	Z	(Prob.)
(1) Sales-Property-Payroll	E		.906	(.943)	--	--	--	--
(2) Sales-Property-Payroll	R		.646	(.645)	-.260	-28.7	3.778	(.000)
(3) Sales-Property	E		.737	(.814)	-.169	-18.7	6.970	(.000)
(4) Sales-Property	R		.737	(.811)	-.169	-18.7	6.705	(.000)
(5) Sales-Payroll	E		1.013	(1.011)	+.107	+11.8	6.941	(.000)
(6) Sales-Payroll	R		.661	(.660)	-.245	-27.0	3.188	(.001)
(7) Property-Payroll	E		.992	(1.000)	+.086	+9.5	7.377	(.000)
(8) Property-Payroll	R		.721	(.731)	-.185	-20.4	3.327	(.001)
(9) Sales	E		.751	(.825)	-.155	-17.1	6.236	(.000)
(10) Property	E		.750	(.784)	-.156	-17.2	4.021	(.000)
(11) Payroll	E		1.282	(1.153)	+.376	+41.5	9.380	(.000)

a $\frac{1}{n} \sum \left| \frac{P-A}{A} \right|$

b Versus formula (1)

c Based on two-tailed Wilcoxon signed ranks

E = equal, R = regression based

TABLE 5

Mean Absolute Error Deflated by Mean Earnings (1979-81)

Statutory Factors	Formula	Weighting	Error ^a		Change ^b		Significance ^c	
			Mean	(CV)	Mean	%	Z	(Prob.)
(1) Sales-Property-Payroll		E	.513	(.979)	--	--	--	--
(2) Sales-Property-Payroll		R	.394	(.900)	-.119	-23.2	3.318	(.001)
(3) Sales-Property		E	.376	(.655)	-.137	-26.7	6.272	(.000)
(4) Sales-Property		R	.375	(.654)	-.138	-26.9	5.808	(.000)
(5) Sales-Payroll		E	.605	(1.171)	+.092	+17.9	5.991	(.000)
(6) Sales-Payroll		R	.414	(1.019)	-.099	-19.3	2.556	(.011)
(7) Property-Payroll		E	.596	(1.221)	+.083	+16.2	6.835	(.000)
(8) Property-Payroll		R	.388	(.655)	-.125	-24.4	2.508	(.012)
(9) Sales		E	.406	(.715)	-.107	-20.9	5.517	(.000)
(10) Property		E	.393	(.724)	-.120	-23.4	3.424	(.001)
(11) Payroll		E	.881	(1.878)	+.368	+71.7	8.614	(.000)

$$^a \frac{1}{n} \sum |P-A| \div \bar{A}$$
^bVersus formula (1)

^cBased on two-tailed Wilcoxon signed ranks

E = equal, R = regression based

TABLE 6

Formula Rankings

Formula		Weighting	Mean Absolute Relative Error ^a				Mean Absolute Error ^b			
Statutory Factors			1979	1980	1981	1979-81	1979	1980	1981	1979-81
(1)	Sales-Property-Payroll	E	5.86	6.08	6.07	6.10	5.88	6.08	6.04	6.05
(2)	Sales-Property-Payroll	R	5.77	5.47	5.66	5.31	5.71	5.47	5.55	5.42
(3)	Sales-Property	E	4.76	4.80	5.10	4.86	4.73	4.80	5.06	4.77
(4)	Sales-Property	R	4.83	4.99	5.28	4.98	4.77	4.99	5.23	4.93
(5)	Sales-Payroll	E	7.09	6.98	6.65	7.13	7.16	6.98	6.71	7.02
(6)	Sales-Payroll	R	5.91	5.80	5.82	5.64	5.92	5.80	5.75	5.81
(7)	Property-Payroll	E	6.72	7.09	6.85	7.05	6.77	7.09	6.99	7.03
(8)	Property-Payroll	R	5.93	5.69	5.73	5.73	5.89	5.69	5.73	5.84
(9)	Sales	E	5.23	4.88	5.11	5.04	5.22	4.88	5.03	4.91
(10)	Property	E	5.50	5.72	5.79	5.63	5.46	5.72	5.81	5.66
(11)	Payroll	E	8.39	8.50	7.94	8.53	8.48	8.50	8.11	8.57
Friedman S			183.7	206.8	119.2	211.0	200.5	206.8	142.4	213.7
(Prob.)			(.000)	(.000)	(.000)	(.000)	(.000)	(.000)	(.000)	(.000)

$$a \frac{1}{n} \sum \left| \frac{P-A}{A} \right| \qquad b \frac{1}{n} \sum \left| \frac{P-A}{A} \right|$$

E = equal, R = regression based

APPENDIX

Statutory Factor Weightings at Segment, State, and Company Levels

Segment to State	$\hat{p}_{ijk} = a_1 \left[\frac{s_{ijk}}{I} \right] + a_2 \left[\frac{c_{ijk}}{I} \right] + a_3 \left[\frac{w_{ijk}}{I} \right]$
State to Company	$\hat{p}_{jk} = a_1 \left[\frac{\frac{I}{\sum s_{ijk}}}{JI} \right] + a_2 \left[\frac{\frac{I}{\sum c_{ijk}}}{JI} \right] + a_3 \left[\frac{\frac{I}{\sum w_{ijk}}}{JI} \right]$
Company to Companies	$\hat{p}_k = a_1 \left[\frac{\frac{JI}{\sum \sum s_{ijk}}}{KJI} \right] + a_2 \left[\frac{\frac{JI}{\sum \sum c_{ijk}}}{KJI} \right] + a_3 \left[\frac{\frac{JI}{\sum \sum w_{ijk}}}{KJI} \right]$

Definitions

\hat{p}_{ijk} = proportion of unitary income attributable to segment i, state j, and company k.

a_i = statutory weights currently set at 1/3 each.

s_{ijk} = sales attributable to segment i, state j, and company k.

c_{ijk} = property attributable to segment i, state j, and company k.

w_{ijk} = payroll attributable to segment i, state j, and company k.

\hat{p}_{jk} = proportion of unitary income attributable to state j and company k.

\hat{p}_k = proportion of unitary income attributable to company k.

UNIVERSITY OF ILLINOIS-URBANA



3 0112 038105364